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Recent results on QCD thermodynamics: lattice QCD versus Hadron Resonance Gas model

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Abstract

We present our most recent investigations on the QCD cross-over transition temperatures with 2+1 staggered flavours and one-link stout improvement [JHEP 1009:073, 2010]. We extend our previous two studies [Phys. Lett. B643 (2006) 46, JHEP 0906:088 (2009)] by choosing even finer lattices (N_t=16) and we work again with physical quark masses. All these results are confronted with the predictions of the Hadron Resonance Gas model and Chiral Perturbation Theory for temperatures below the transition region. Our results can be reproduced by using the physical spectrum in these analytic calculations. A comparison with the results of the hotQCD collaboration is also discussed.

1. Introduction

One of the most interesting quantities that can be extracted from lattice simulations is the transition temperature T_c at which hadronic matter is supposed to undergo a transition to a deconfined, quark-gluon phase. This quantity has been vastly debated over the last few years, due to the disagreement on its numerical value observed by different lattice collaborations, which in some cases is as high as 20% of the absolute value.

Indeed, the analysis of the hotQCD collaboration (performed with two different improved staggered fermion actions, asqtad and p4, and with physical strange quark mass and somewhat larger than physical u and d quark masses, $m_s/m_{u,d} = 10$), indicates that the transition region lies in the range $T = (185-195)$ MeV (for the latest published result and for references see [1]). The results obtained by our collaboration using the staggered stout action (with physical light and strange quark masses, thus $m_s/m_{u,d} \simeq 28$) show that the value of the transition temperature lies in the range 150-170 MeV, and it changes with the observable used to define it [2, 3]. This reflects the cross-over nature of the transition [4]. We present our most recent results for several physical quantities (for all details see [5]): our previous works [2, 3] have been extended to an even smaller lattice spacing (down to $a \lesssim 0.075$ fm at T_c), corresponding to N_t=16. We use physical light and strange quark masses: we fix them by reproducing f_K/m_π and f_K/m_K and by this procedure [3] we get $m_s/m_{u,d} = 28.15$. We also present some aspects of the Hadron Resonance Gas model and the comparison between HRG model results and the lattice data from ours and the hotQCD collaborations[1]. As we will see, our analysis provides a straightforward explanation for the observed discrepancy in the results of the two collaborations.

[1]Note, that recently preliminary results were presented [6] and the results of the hotQCD collaboration moved closer to our results.

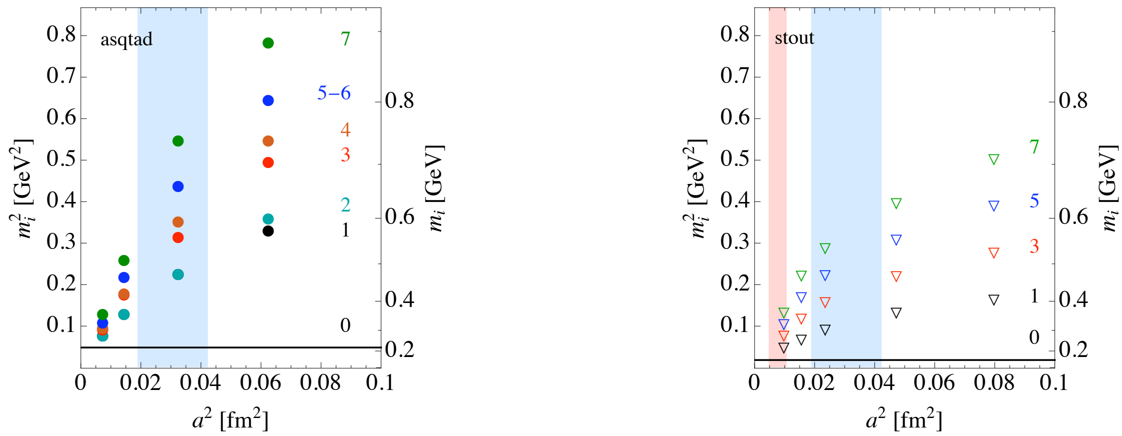

Figure 1: Masses of the pion multiplet squared, as functions of the lattice spacing squared. Left panel: asqtad action [18]. Right panel: stout action. The numbers next to the data correspond to the taste matrices (see Ref. [5] for details). In both panels, the blue band indicates the relevant range of lattice spacings for a thermodynamics study at $N_t = 8$ between $T = 120$ and 180 MeV. The red band in the right panel corresponds to the same temperature range and $N_t = 16$. In both figures, the horizontal line labelled as "0" is the pseudo-Goldstone boson, which has a mass of 220 MeV for the asqtad results, and 135 MeV for the stout ones

2. Details of the lattice simulations

We use [2, 3] a tree-level Symanzik improved gauge, and a stout-improved staggered fermionic action (see Ref. [7] for details). The stout-smearing is an important part of the framework, which reduces the taste violation. Indeed, in [8] we pointed out that the continuum limit can be approached only if one reduces the unphysical pion splitting (the main motivation of our choice of action).

The pion splitting of a staggered framework has to vanish in the continuum limit. Once it shows an $\alpha_s a^2$ dependence (in practice a^2 dependence with a subdominant logarithmic correction) we are in the scaling region. This is an important check for the validity of the staggered framework at a given lattice spacing. In Fig. 1 we show the leading order a^2-behavior of the masses of the pion multiplets calculated with the asqtad (left) and stout (righ) actions. It is evident that the continuum expectation is reached faster in the stout action than in the asqtad one. In addition, in the present paper we push our results to $N_t = 16$, which corresponds to even smaller lattice spacings and mass splittings than those used in [3].

In analogy with what we did in [2, 3], we set the scale at the physical point by simulating at $T = 0$ with physical quark masses [3] and reproducing the kaon and pion masses and the kaon decay constant. This gives an uncertainty of about 2% in the scale setting, which propagates in the uncertainty in the determination of the temperature values.

3. Lattice vs Hadron Resonance Gas model results

We present our lattice results for the strange quark number susceptibility, chiral condensate and interaction measure. We extract the values of the transition temperature associated to these observables. We also perform a HRG analysis and compare the HRG predictions to our results and to those of the hotQCD Collaboration.

The Hadron Resonance Gas model has been widely used to study the low temperature phase of QCD in comparison with lattice data. In Ref. [9] an important ingredient was included in this model, namely the pion mass- and lattice spacing-dependence of the hadron masses. This gives rise to a resonance spectrum which is distorted by lattice artifacts, and which needs to be taken into account in the comparison with the results of the hotQCD collaboration. Here we combine these ingredients with Chiral Perturbation Theory (χPT) [10]. This opens the possibility to study chiral quantities, too. The HRG model has its roots in the theorem by Dashen, Ma and Bernstein [11], which allows to calculate the microcanonical partition function of an interacting system, in the thermodynamic limit $V \to \infty$, to a good approximation, assuming that it is a gas of non-interacting free hadrons and resonances [12]. The pressure of the Hadron Resonance Gas model can be written as the sum of independent contributions coming from non-interacting

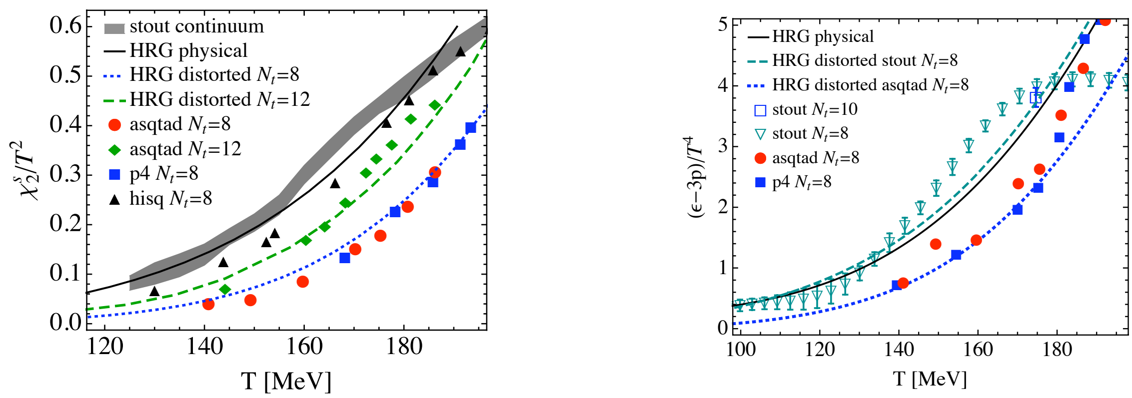

Figure 2: Left panel: strange quark susceptibility as a function of the temperature. full symbols correspond to results obtained with the asqtad, p4 and hisq actions [1, 6]. Our continuum result is indicated by the gray band. The solid line is the HRG model result with physical masses. The dashed and dotted lines are the HRG model results with distorted masses corresponding to $N_t = 12$ and $N_t = 8$, which take into account the discretization effects and heavier quark masses, which characterize the results of the hotQCD Collaboration. Right panel: $(\epsilon - 3p)/T^4$ as a function of the temperature. Open symbols are our results. Full symbols are the results for the asqtad and p4 actions at $N_t = 8$ [1]. Solid line: HRG model with physical masses. Dashed lines: HRG model with distorted spectrums. As it can be seen, the prediction of the HRG model with a spectrum distortion corresponding to the stout action at $N_t = 8$ is already quite close to the physical one. The error on the recent preliminary HISQ result [6] is larger than the difference between the stout and asqtad data, that is why we do not show them here.

resonances. We include all known baryons and mesons up to 2.5 GeV, as listed in the latest edition of the Particle Data Book (for an improvement of the model by including an exponential mass spectrum see [13]). We will compare the results obtained with the physical hadron masses to those obtained with the distorted hadron spectrum which takes into account lattice discretization effects. Each pseudoscalar meson in the staggered formulation is split into 16 mesons with different masses, which are all included. Similarly to Ref. [9], we will also take into account the pion mass- and lattice spacing- dependence of all other hadrons and resonances.

Quark number susceptibilities increase during the transition, therefore they can be used to identify this region. They are defined as $\chi_2^q = \frac{T}{V}\frac{\partial^2 \ln Z}{\partial(\mu_q)^2}\Big|_{\mu_i=0}$, (with $q = u, d, s$). In the left panel of Fig. 2 we show our continuum-extrapolated results for the strange quark number susceptibility, in comparison with the HRG results with physical spectrum. Also shown are the hotQCD collaboration data, in comparison with the HRG model results with distorted spectrum. In the right panel of Fig. 2 we show the trace anomaly $(\epsilon - 3p)$ divided by T^4 as a function of the temperature. Our $N_t = 8$ results are taken from Ref. [14]. Notice that, for this observable, we have a check-point at $N_t = 10$: the results are on top of each other. Also shown are the results of the hotQCD collaboration at $N_t = 8$ [1] and the HRG model predictions for physical and distorted resonance spectrums. On the one hand, our results are in good agreement with the "physical" HRG model ones. It is important to note, that using our mass splittings and inserting this distorted spectrum into the HRG model gives a temperature dependence which lies essentially on the physical HRG curve (at least within our accuracy). On the other hand, a distorted spectrum based on the asqtad and p4 frameworks results in a shift of about 20 MeV to the right.

In order to compare our results to those of the hotQCD collaboration, we also calculate the quantity $\Delta_{l,s} = (\langle\bar{\psi}\psi\rangle_{l,T} - \frac{m_l}{m_s}\langle\bar{\psi}\psi\rangle_{s,T})/\langle\bar{\psi}\psi\rangle_{l,0} - \frac{m_l}{m_s}\langle\bar{\psi}\psi\rangle_{s,0})$ (with $l = u, d$). We compare our results to the predictions of the HRG model and χPT [15]. To this purpose, we need to know the quark mass dependence of the masses of all resonances included in the partition function. We assume that all resonances behave as their fundamental states as functions of the quark mass, and take this information from Ref. [16]. They agree with the results obtained by our collaboration in [17].

4. Conclusions

We have presented our latest results for the QCD transition temperature. The quantities that we have studied are the strange quark number susceptibility, the chiral condensate and the trace anomaly. We have given the complete temperature dependence of these quantities, which provide more information than the characteristic temperature val-

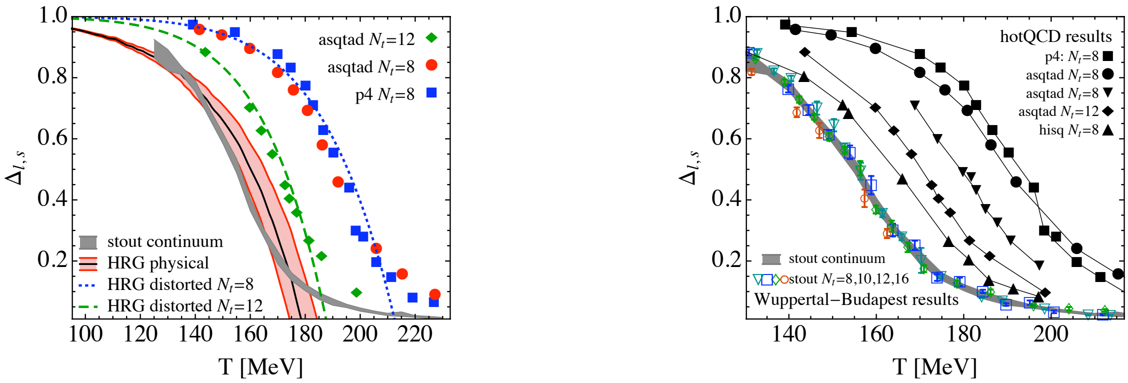

Figure 3: Subtracted chiral condensate $\Delta_{l,s}$ as a function of the temperature. Gray bands are the continuum results of our collaboration, obtained with the stout action. Full symbols are obtained with the asqtad and p4 actions [1, 6]. In the left panel, the solid line is the HRG model+χPT result with physical masses. The error band corresponds to the uncertainty in the quark mass-dependence of hadron masses. The dashed lines are the HRG+χPT model results with distorted masses, which take into account the discretization effects and heavier quark masses used in [1, 6] for $N_t = 8$ and $N_t = 12$. In the right panel we show a comparison between stout, asqtad, p4 and HISQ results. Our results are shown by colored open symbols, whereas the hotQCD results are shown by full black symbols. The gray band is our continuum result, the thin lines for the hotQCD data are intended to lead the eye. Our stout results were all obtained by the physical pion mass of 135 MeV. The full dots and squares correspond to $m_\pi = 220$ MeV, the full triangles and diamonds correspond to $m_\pi = 160$ MeV of the hotQCD collaboration.

ues alone. Our previous results have been pushed to an even finer lattice ($N_t = 16$). The new data corresponding to $N_t = 16$ confirm our previous results. The trace anomaly [14] was obtained for $N_t = 8$ and a check-point at $N_t = 10$. The transition temperature that we obtain from this last quantity is very close to the one obtained from the chiral condensate. In order to find the origin of the discrepancy between the results of our collaboration and the hotQCD ones, we calculated these observables in the Hadron Resonance Gas model. Besides using the physical hadron masses, we also performed the calculation with modified masses which take into account the heavier pions and larger lattice spacings used in [1]. We find an agreement between our data and the HRG ones with "physical" masses, while the hotQCD collaboration results are in agreement with the HRG model once the spectrum is "distorted" as it was directly measured on the lattice [18]. All the details can be found in Ref. [5].

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